

Session 4:

Design Recommendations: Strut-and-Tie Method and some reconciliations with rebar and anchor theory

Dr. Daniel Looi

PhD (HKU) | BEng (Malaya)

Lecturer | Swinburne University of Technology (Sarawak Malaysia)

dlooi@swinburne.edu.my



24 September 2019

Content

1. Challenges of PIR design
2. Recommendation 1: Strut-and-tie method for strut check
3. Recommendation 2: Strut-and-tie method for tie force
4. Recommendation 3: Option for bond stress
5. Recommendation 4: Minimum cover and edge distance
6. Design example
7. Reconciliation with BA theory
8. Conclusion

1. Challenges of PIR design

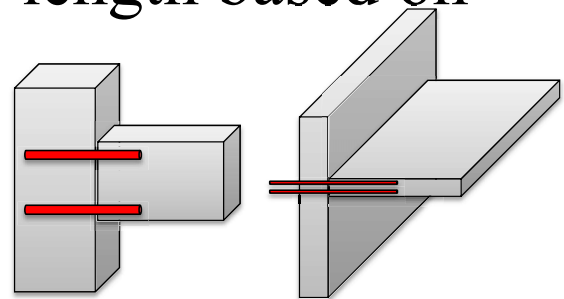
- The two distinct theories (namely **REA theory as per EN 1992-1-1, 2004** and **BA theory as per EN1992-4, 2018**) were developed individually with departed philosophy.
- **REA theory** for cast-in rebars is ideal for most engineers due to familiarity. However, engineers may find the computed **anchorage length can be overly long**.

3

An example of anchorage length based on EN 1992-1-1 (2004)

$$f_{cu,k} = 30 \text{ MPa}, f_{yk} = 500 \text{ Mpa}$$

$$\alpha_2 = 0.7, \alpha_{lb} = 1.5$$



ϕ (mm)	$l_b = \alpha_2 l_{b,rd} \geq l_{min}$ in EC2 (mm)	l_b in BS 8110 (mm)	Remarks on beam-column connection	Remarks on slab-wall connection
12	338	476	Constructible, provided the column sectional depth is sufficient.	More critical than beam- column connections, due to the limited thickness of the wall.
16	451	635		
20	564	794		
25	705	992		
32	902	1270		

4

The challenges of BA theory

- Very **short anchorage length** – uncommon in PIR practice and not thoroughly researched although there were technical papers (*Mahrenholtz et al., 2015 and Charney et al. 2013*) proposed BA for PIR.
- More common in concrete-to-steel connection, rather than concrete-concrete connection.

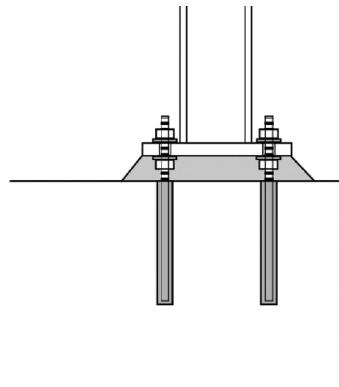


Figure taken from:

Charney et al. (2013). Recommended Procedures for Development and Splicing of Post-Installed Bonded Reinforcing Bars in Concrete Structures. *ACI Structural Journal*, 110(3), 437-446

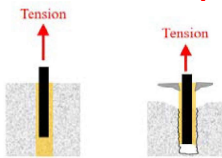
Fig. 1—Adhesive anchors used to secure column baseplate.

The challenges of BA theory

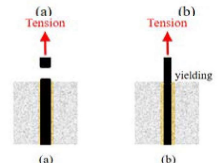
- Should the capacity based on cracked or uncracked concrete?
Some technical discussions can be found at

<http://www.aefac.org.au/documents/AEFAC-TN06-concrete.pdf>

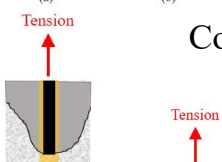
- **Complex computations** with many coefficient factors.



Steel failure (Cl. 6.2.2) $N_{Rd,s} = f_{uk} A_s / \gamma_{Ms}$ where $\gamma_{Ms} = 1.2$ $f_{uk} / f_{yk} \geq 1.4$
 $N_{Rd,y} = f_{yk} A_s / \gamma_s$ where $\gamma_s = 1.15$



Combined bond (pull-out) and concrete failure (Cl. 6.2.2) $N_{Rk,p} = f_{bd} \pi \phi l_b$
 $N_{Rk,p} = N_{Rk,p}^{\circ} \psi_{A,Np} \psi_{s,Np} \psi_{g,Np} \psi_{re,N} \psi_{ec,Np}$



Concrete cone (breakout) failure (Cl. 6.2.3) $N_{Rk,c} = k_{cr} \sqrt{f_{cu}} l_b^{1.5}$
 $N_{Rk,c} = N_{Rk,c}^{\circ} \psi_{A,Nc} \psi_{s,N} \psi_{re,N} \psi_{ec,N}$



Splitting failure (Cl. 6.2.4) $N_{Rk,sp} = N_{Rk,c}^{\circ} \psi_{A,Nsp} \psi_{s,Nsp} \psi_{re,N} \psi_{ec,N} \psi_{h,sp}$

The challenges of BA theory

- Additional check for **shear resistance**
- Interaction check of **tension + shear**
- Due to the complexity of the process, many manufacturers offer **software** that performs this task.

Recommendations for PIR design

- In view of the challenges, **4 design recommendations** are proposed.
- A **design example** is illustrated with the use of the recommendations.

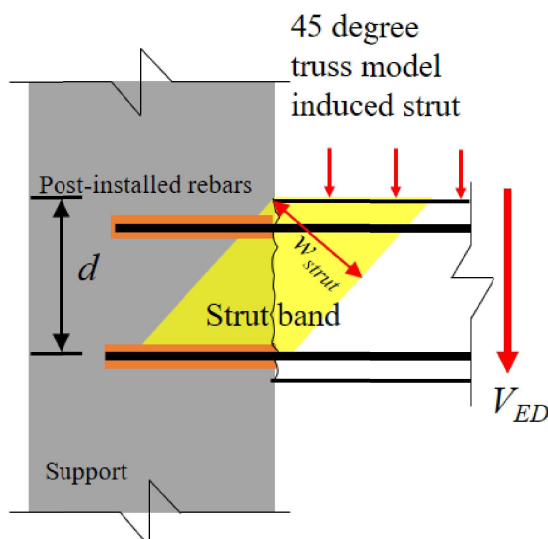
Content

1. Challenges of PIR design
2. Recommendation 1: Strut-and-tie method for strut check
3. Recommendation 2: Strut-and-tie method for tie force
4. Recommendation 3: Option for bond stress
5. Recommendation 4: Minimum cover and edge distance
6. Design example
7. Reconciliation with BA theory
8. Conclusion

9

Recommendation 1: STM for strut check

- Use STM to check strut strength to avoid web crushing failure



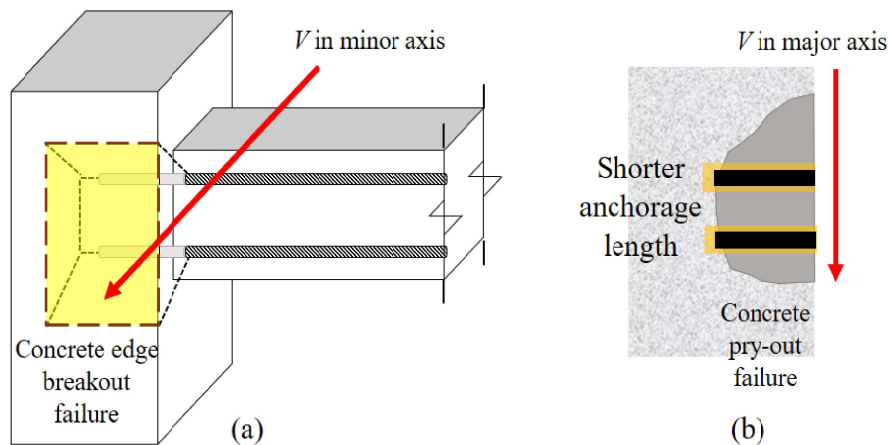
$$w_{strut} = d/\sqrt{2}$$

$$f_{strut} = \frac{V_{Ed}}{b w_{strut}} \leq v_{strut} f_{cu,k}/\gamma_m$$

10

Recommendation 1: STM for strut check

- (a) No out-of-plane shear (V in minor axis)
- (b) Provide minimum anchorage length to preclude concrete pry-out failure (V in major axis).



11

Content

1. Challenges of PIR design
2. Recommendation 1: Strut-and-tie method for strut check
3. Recommendation 2: Strut-and-tie method for tie force
4. Recommendation 3: Option for bond stress
5. Recommendation 4: Minimum cover and edge distance
6. Design example
7. Reconciliation with BA theory
8. Conclusion

12

Recommendation 1: STM for tie force

- Use STM to compute the **actual acting force**, rather than using the **yield strength of steel**.
- In EN 1992-1-1 (2004), the design stress (σ_{sd}) is not precisely described in the code.
- An article written by the Concrete Centre of the Mineral Products Association (MPA) (CDG-5, 2015) stated that σ_{sd} can be **rationally determined using the ratio of steel area required ($A_{s,rqd}$) to steel area provided ($A_{s,prov}$), multiply by the design yield strength of steel (i.e., $A_{s,rqd}/A_{s,prov} \cdot f_{yk}/\gamma_s$), but still pretty much **relying on the yield strength**.**

13

STM in EN 1992-1-1 (2004)

- Cl. 9.2.1.4(2) allows a **STM** to calculate the axial forces (F_{Ed}) in the rebar, which suits well to estimate the design stress (σ_{sd})

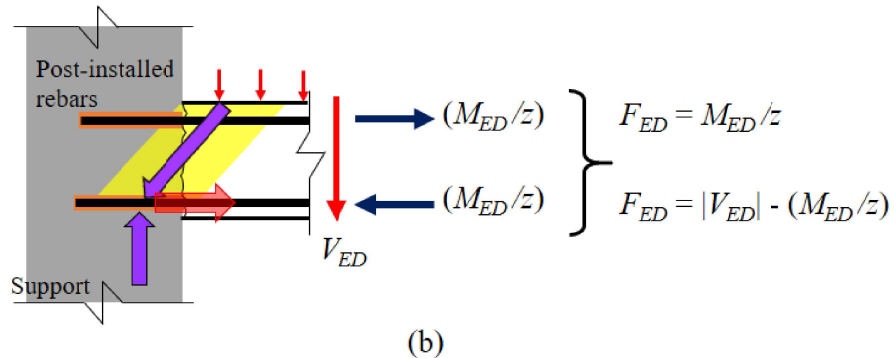
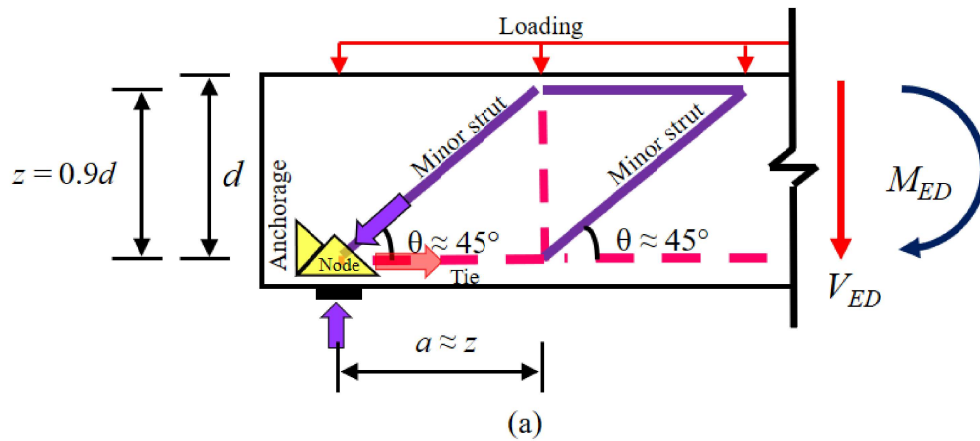
$$F_{Ed} = |V_{Ed}| \frac{a}{z} + N_{Ed}$$

Where, V_{Ed} is the design shear force, a is the shear span, z is assumed to be $0.9 d$, d is the effective depth of the section and N_{Ed} is the axial force (direct axial or resulted from bending) to be added to or subtracted from the tensile force.

$$\sigma_{sd} = F_{Ed}/A_s = [|V_{Ed}| \pm M_{Ed}/z] / A_s$$

14

STM



$$\sigma_{sd} = F_{Ed}/A_s = [|V_{Ed}| \pm M_{Ed}/z] / A_s$$

15

Some notes for simply supported members

- **zero tension** at the top bar for simply supported member is an **idealised assumption**.
- This assumption should be **reviewed** based on the provided top bar as per the **minimum rebar percentage** (i.e., **0.13% A_c**) and the **partial fixity detailing practice** (i.e., Cl. 9.3.1.2(2) of EN 1992-1-1 (2004) recommended that end support moment to be resisted may be reduced to 15% of the maximum moment in the adjacent span for slab, to be resisted by the top bar.)

16

Content

1. Challenges of PIR design
2. Recommendation 1: Strut-and-tie method for strut check
3. Recommendation 2: Strut-and-tie method for tie force
4. Recommendation 3: Option for bond stress
5. Recommendation 4: Minimum cover and edge distance
6. Design example
7. Reconciliation with BA theory
8. Conclusion

17

Recommendation 3: Option for bond stress

Provide flexibility for engineers by having: -

- **Option 1:** f_{bu} as per EN 1992-1-1 (2004) cast-in rebar, hence α_2 as per **EC2**;

$$0.7 \leq \alpha_2 = 1 - \frac{0.15(c_d - \phi)}{\phi} \leq 1.0 \text{ (Tension)}$$

- **Option 2:** f_{bu} as per ETA or manufacturer's technical data, hence α_2' as per **extended EC2 method** for higher bond stress.

$$\alpha_2' = \frac{1}{\frac{1}{0.7} + \delta \frac{c_d - 3\phi}{\phi}} \geq 0.25 \text{ (Tension)}$$

18

Content

1. Challenges of PIR design
2. Recommendation 1: Strut-and-tie method for strut check
3. Recommendation 2: Strut-and-tie method for tie force
4. Recommendation 3: Option for bond stress
5. Recommendation 4: Minimum cover and edge distance
6. Design example
7. Reconciliation with BA theory
8. Conclusion

19

Recommendation 4: Minimum cover and edge distance

- This recommendation is to account for **splitting failure**.
- EN 1992-1-1 (2004) stated that the maximum boundary is reached when α_2 equals to 1.0, c_d corresponds to 1ϕ . It should be noted that such small cover of 1ϕ may present **challenges in hole drilling** for post-installed rebar system.

20

Recommendation 4: Minimum cover and edge distance

- EOTA EAD 330087 (2018) proposed the minimum cover as a function of **drilling method**, rebar size and with or without the use of drilling aid, to take into account the possible deviations during the drilling process.

Use of drilling aid	Drilling method	Bar diameter ϕ	c_{\min}
No	Hammer or diamond	< 25 mm	$30 \text{ mm} + 0.06 l_v \geq 2\phi$
		≥ 25 mm	$40 \text{ mm} + 0.06 l_v \geq 2\phi$
	Compressed air	< 25 mm	$50 \text{ mm} + 0.08 l_v$
		≥ 25 mm	$60 \text{ mm} + 0.08 l_v \geq 2\phi$
Yes	Hammer or diamond	< 25 mm	$30 \text{ mm} + 0.02 l_v \geq 2\phi$
		≥ 25 mm	$40 \text{ mm} + 0.02 l_v \geq 2\phi$
	Compressed air	< 25 mm	$50 \text{ mm} + 0.02 l_v$
		≥ 25 mm	$60 \text{ mm} + 0.02 l_v \geq 2\phi$

where l_v is the setting anchorage depth of rebars (in unit mm).

21

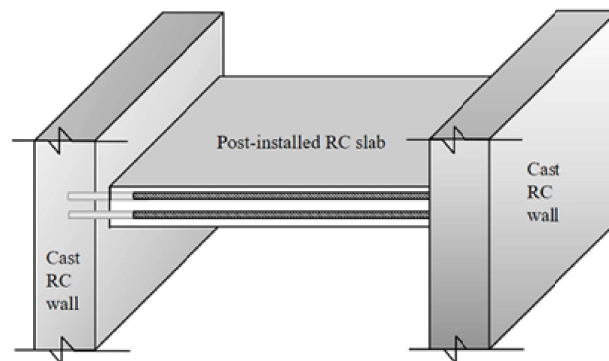
Content

- Challenges of PIR design
- Recommendation 1: Strut-and-tie method for strut check
- Recommendation 2: Strut-and-tie method for tie force
- Recommendation 3: Option for bond stress
- Recommendation 4: Minimum cover and edge distance
- Design example
- Reconciliation with BA theory
- Conclusion

22

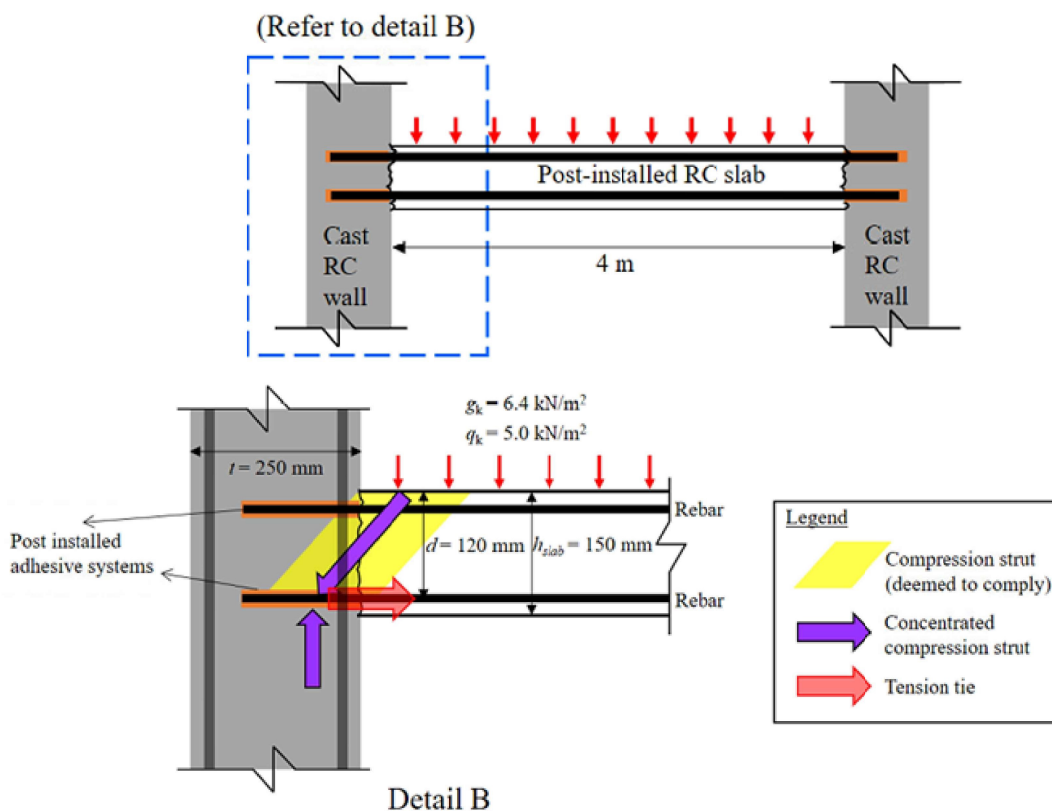
Detail design example - A simply supported RC slab connected to an RC shear wall – 1/15

- During the execution of construction, the RC slab is planned to be cast after the construction of the RC shear wall. No starter bar was pre-embedded; hence post-installed rebar is considered. The post-installed rebar for a new RC slab is to be designed.



23

Design example – 2/15



24

Design example – 3/15

Structure dimension, material and load

- **Slab:** $l_n = 4 \text{ m}$, $h_{\text{slab}} = 150 \text{ mm}$, $b = 1000 \text{ mm}$ (for per metre run),
cover = 30 mm, $d = 120 \text{ mm}$, $a_v = d$
- **Shear wall:** $h_{\text{wall}} = 250 \text{ mm}$, cover = 50 mm, $\phi 25$ vertical and horizontal bar
at 250 mm spacing
- **Concrete grade:** C35 (cube), $f_{\text{ctk},0.05} \approx 1.95 \text{ MPa}$
- **Reinforcement:** $f_{\text{yk}} = 500 \text{ N/mm}^2$, $\gamma_s = 1.15$

- **Permanent actions / Dead loads (self-weight):**
 $g_k = 24.5 \text{ kN/m}^3 \times h = 24.5 \times 0.15 = 3.7 \text{ kN/m}^2$
- **Permanent actions / SDL (screeding, tiles, electrical, partition walls):**
 $g_k = 2.7 \text{ kN/m}^2$
- **Variable actions / Live loads:** $q_k = 5 \text{ kN/m}^2$
- **Actions combination:** At ULS, $S_d = (1.35 g_k + 1.50 q_k) = 16.1 \text{ kN/m}^2$

25

Design example – 4/15

Structural analysis (design forces):

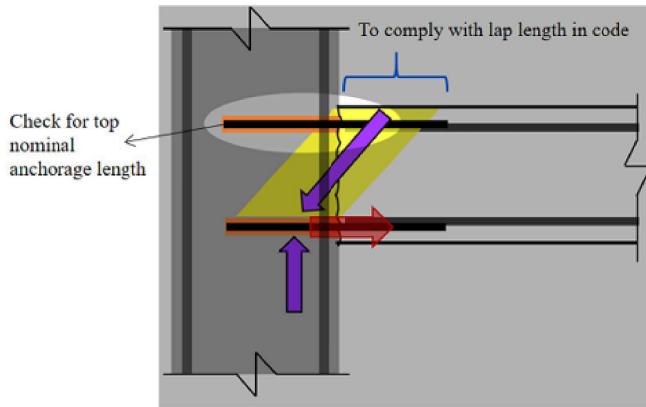
- At mid span, $M_{\text{Ed}} = S_d l_n^2 / 8 = 32.2 \text{ kNm/m}$
- At support, $V_{\text{Ed}} = S_d l_n / 2 = 32.2 \text{ kN/m}$

Pre-designed slab

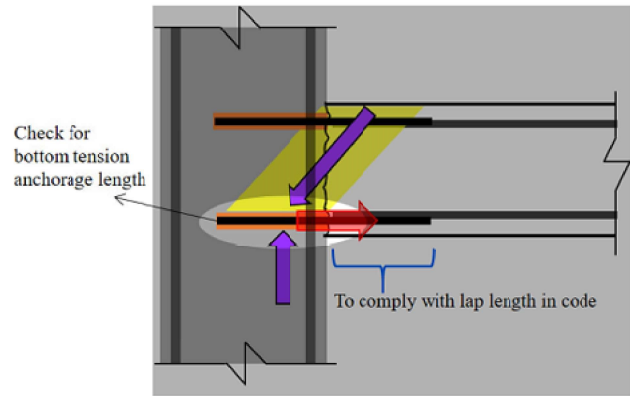
- Bottom reinforcement required:
At mid span, $A_{\text{s,rqd,m}} = M_{\text{Ed}} / (0.9d f_{\text{yk}} / \gamma_s) = 686 \text{ mm}^2/\text{m}$
- Reinforcement provided:
At mid span, $\phi 10$, $s = 100 \text{ mm}$; $A_{\text{s,prov,m}} = 785 \text{ mm}^2/\text{m}$

26

Design example – 5/15



(a) Top (for nominal)



(b) Bottom (for tension)

Design example – 6/15

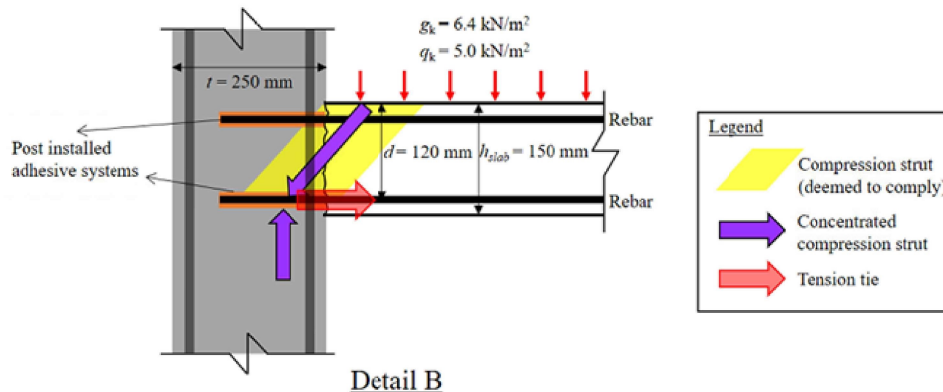
Minimum post-installed reinforcement to be anchored at support

$A_{s,min}$ is generally = $0.13\% A_c = 195 \text{ mm}^2/\text{m}$

Top	Bottom
<p>Simplified rules Cl. 9.3.1.2(2) of EN 1992-1-1 (2004) (i.e., 15% of the maximum bending in the span) for simply supported slab to control cracking at negative moments due to partial fixity:</p>	<p>At the end support of simply-supported slab or continuous slab, half the calculated mid-span bottom reinforcement should be anchored:</p>
$A_{s,simplified\ rules} = 0.15 M_{Ed, mid-span} / (0.9d f_{yk}/\gamma_s) = 103 \text{ mm}^2/\text{m}$	$A_{s,simplified\ rules} = 0.50 A_{s,mid-span} = 0.50 (686) = 343 \text{ mm}^2/\text{m}$
<p>Provide $\phi 10@200$ ($393 \text{ mm}^2/\text{m}$)</p>	<p>Provide $\phi 10@200$ ($393 \text{ mm}^2/\text{m}$)</p>

Design example – 7/15

Recommendation 1: STM for strut check



- Assuming a 45° strut relative to the bottom longitudinal bar
- $F_{strut} = |V_{Ed}| / \sin 45^\circ = 46 \text{ kN/m}$
- Strut width, $w_{strut} = h_{slab} / \sqrt{2} = 150 / \sqrt{2} = 106 \text{ mm}$.
- $f_{strut} = \frac{F_{strut}}{bw_{strut}} = 0.43 \text{ MPa}$
 $< 0.60 f_{cu} / 1.5 = 14 \text{ MPa}$, hence OK.

Su, R.K.L and Looi, D.T.W. (2016). "Revisiting the Unreinforced Strut Efficiency Factor", ACI Structural Journal, 113(2), pp. 301-312.

29

Design example – 8/15

Recommendation 2: STM for tie force

Top	Bottom
<p>Shear (V_{Ed}) will not transfer tension to the top bar and end moment is zero due to simply supported assumption: $\sigma_{sd} = F_{Ed} / A_s = [V_{Ed} \pm M_{Ed} / z] / A_s = 0$</p>	<p>Shear (V_{Ed}) will induce a direct tension via a strut of 45-degree to the bottom bar. The end moment is zero due to simply supported assumption: $\sigma_{sd} = F_{Ed} / A_s = [V_{Ed} \pm M_{Ed} / z] / A_s$ Hence, $F_{Ed,tens} = V_{Ed} + 0 = 32 \text{ kN/m}$</p> <p>Post-installed steel area required: $A_{s,rqd} = F_{Ed} / (f_{yk} / \gamma_s) = 74 \text{ mm}^2/\text{m}$</p>

30

Design example – 9/15

Recommendation 3: Option for bond stress

The top and bottom supports are in tension

- **Option 1:** f_{bu} as per EN 1992-1-1 (2004) cast-in rebar, hence α_2 as per EC2;

$$\alpha_2 = 0.85;$$

$$f_{bu} = 2.25 \eta_1 \eta_2 f_{ctk,0.05} / \gamma_m / \alpha_2 = 3.4 \text{ MPa}$$

- **Option 2:** f_{bu} as per ETA or manufacturer's technical data, hence α_2' as per extended EC2 method for higher bond stress.

$$cd = 50 \text{ mm}; \phi = 10 \text{ mm}; \delta = 0.15;$$

$$\alpha_2' = \frac{1}{\frac{1}{0.7} + \delta \frac{c_d - 3\phi}{\phi}} \geq 0.25 = 0.4;$$

$$f_{bu} = 2.25 \eta_1 \eta_2 f_{ctk,0.05} / \gamma_m / \alpha_2 = 7.3 \text{ MPa}$$

31

Design example – 10/15

Calculation of required anchorage length

Top	Bottom
<p>Back-calculate the equivalent hogging moment at support:</p> $A_{s,req} \text{ top support} / A_{s,bottom} \text{ mid span} \times M_{Ed} =$ $195/686 \times 32.2 = 9.2 \text{ kNm/m}$ <p>Equivalent stress:</p> $\sigma_{sd} = [9.2 / (0.9 d)] \times 1000 / (5 \pi 10^2/4)$ $= 216 \text{ MPa}$ <p><u>Option 1</u></p> $l_{b,rqd} = \frac{\sigma_{sd} \phi}{f_{bd} 4} = \frac{216}{3.4} \left(\frac{10}{4}\right) = 159 \text{ mm}$ <p><u>Option 2</u></p> $l_{b,rqd} = \frac{\sigma_{sd} \phi}{f_{bd} 4} = \frac{216}{7.3} \left(\frac{10}{4}\right) = 74 \text{ mm}$	<p>From STM, the pull-out tension is equivalent to the shear force $F_{Ed,tens} = 32.2$ kN/m</p> $\sigma_{sd} = 32200 / (5 \pi 10^2/4) = 82 \text{ MPa}$ <p><u>Option 1</u></p> $l_{b,rqd} = \frac{\sigma_{sd} \phi}{f_{bd} 4} = \frac{82}{3.4} \left(\frac{10}{4}\right) = 60 \text{ mm}$ <p><u>Option 2</u></p> $l_{b,rqd} = \frac{\sigma_{sd} \phi}{f_{bd} 4} = \frac{82}{7.3} \left(\frac{10}{4}\right) = 28 \text{ mm}$

32

Design example – 11/15

Calculation of required anchorage length (with yield strength)

Top	Bottom
If design with $f_y = 500$ MPa	If design with $f_y = 500$ MPa
$l_{b,rqd} = \frac{0.87 f_y \phi}{f_{bd}} \frac{\phi}{4} = \frac{0.87(500) 10}{3.4} \frac{10}{4}$ $= 320 \text{ mm}$	$l_{b,rqd} = \frac{0.87 f_y \phi}{f_{bd}} \frac{\phi}{4} = \frac{0.87(500) 10}{3.4} \frac{10}{4}$ $= 320 \text{ mm}$

Note that the anchorage length calculated using yield strength has already penetrated the RC walls thickness of 250 mm

33

Design example – 12/15

Calculation of minimum anchorage length

Top	Bottom
$l_{b,min} \geq \max\{0.3l_{b,req}; 10\phi; \alpha_{lb} 100 \text{ mm}\}$	$l_{b,min} \geq \max\{0.3l_{b,req}; 10\phi; \alpha_{lb} 100 \text{ mm}\}$
<u>Option 1</u> $l_{b,min} \geq \max\{0.3(159) = 48; 10(10) =$	<u>Option 1</u> $l_{b,min} \geq \max\{0.3(60) = 18; 10(10) =$

34

Design example – 13/15

Provide anchorage length

Top	Bottom
<u>Option 1</u> $l_b = \max \{l_{b,req}, l_{b,min}\} = \max \{159, 150\} = 160 \text{ mm}$	<u>Option 1</u> $l_b = \max \{l_{b,req}, l_{b,min}\} = \max \{60, 150\} = 150 \text{ mm}$
<u>Option 2</u> $l_b = \max \{l_{b,req}, l_{b,min}\} = \max \{74, 150\} = 150 \text{ mm}$	<u>Option 2</u> $l_b = \max \{l_{b,req}, l_{b,min}\} = \max \{28, 150\} = 150 \text{ mm}$

35

Design example – 14/15

Recommendation 4: Minimum cover and edge distance

Top	Bottom
$c_d = \min \{s/2, c_1, c\} = \min \{100/2, 50, 50\} = 50 \text{ mm}$	$c_d = \min \{s/2, c_1, c\} = \min \{100/2, 50, 50\} = 50 \text{ mm}$
Apply drilling aid, compressed air drilled, $\phi = 10$, hence $50 \text{ mm} + 0.02 l_v$	Apply drilling aid, compressed air drilled, $\phi = 10$, hence $50 \text{ mm} + 0.02 l_v$
<u>Option 1</u> $50 + 0.02 (160) = 53 \text{ mm}$	<u>Option 1 and 2</u> $50 + 0.02 (150) = 53 \text{ mm}$
<u>Option 2</u> $50 + 0.02 (150) = 53 \text{ mm}$	

36

Design example – 15/15

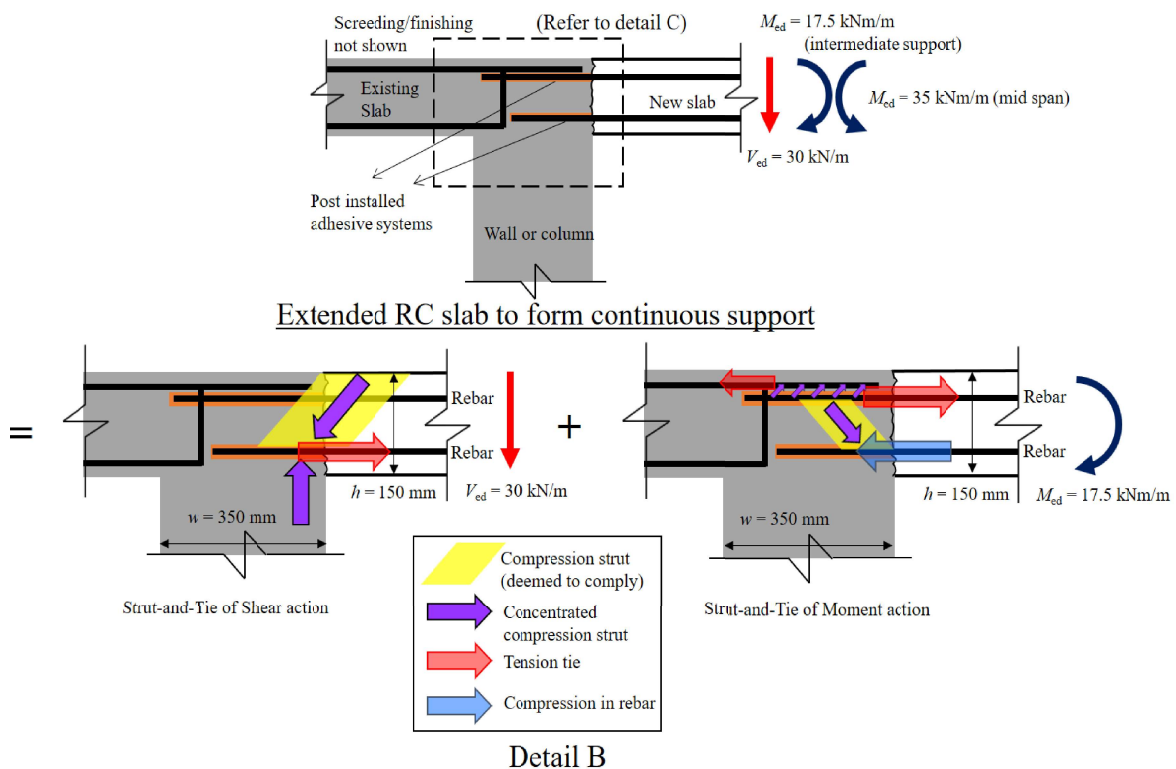
Summary

Top	Bottom
Hence provide: 5 T10 @ 100 mm, $A_{s,prov} = 393 \text{ mm}^2$	Hence provide: 5 T10 @ 100 mm, $A_{s,prov} = 393 \text{ mm}^2$
$l_b = 160 \text{ mm}$ (Option 1) or $l_b = 150 \text{ mm}$ (Option 2)	$l_b = 150 \text{ mm}$ (Option 1 and 2)
$c_d = 55 \text{ mm}$.	$c_d = 55 \text{ mm}$.

Since l_b extends more than the centreline of the support ($250/2 = 125 \text{ mm}$), hence need NOT to check for additional moment induced by the eccentricity on the support.

37

Brief example: Decoupling of moment connection



38

Content

1. Challenges of PIR design
2. Recommendation 1: Strut-and-tie method for strut check
3. Recommendation 2: Strut-and-tie method for tie force
4. Recommendation 3: Option for bond stress
5. Recommendation 4: Minimum cover and edge distance
6. Design example
7. Reconciliation with BA theory
8. Conclusion

39

Reconciliation with BA theory for the detail design example

Summary of design information:

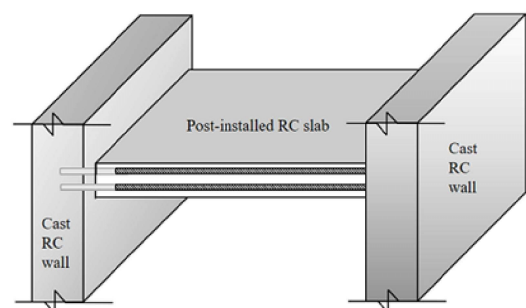
- Base concrete: 250 mm thick wall, $c_d = 50$ mm, C35 cube strength
- PIR: bond strength $f_{bd} = 7.3$ MPa, use T10 @ 200 (5 bars per m run)
- Load: $32 \text{ kN/m} / 5 = 6.4 \text{ kN per bar}$

Reconciliation with BA theory for:

- Uncracked / cracked condition

See graphs on next slides

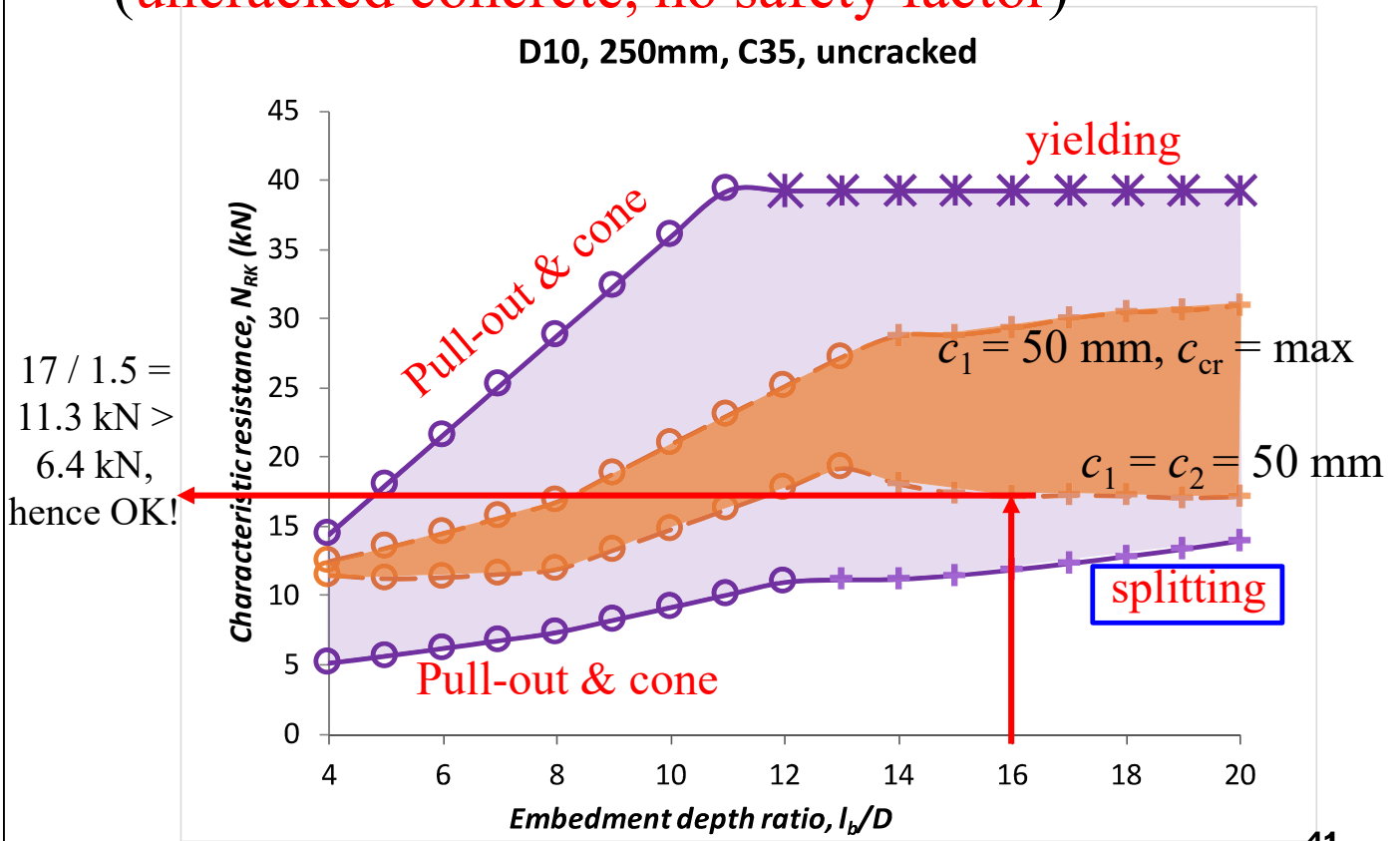
Acknowledgment: The computation work of BA theory was done by Ms. Eva Wong Shu Wen, graduate of Swinburne University (Sarawak)



40

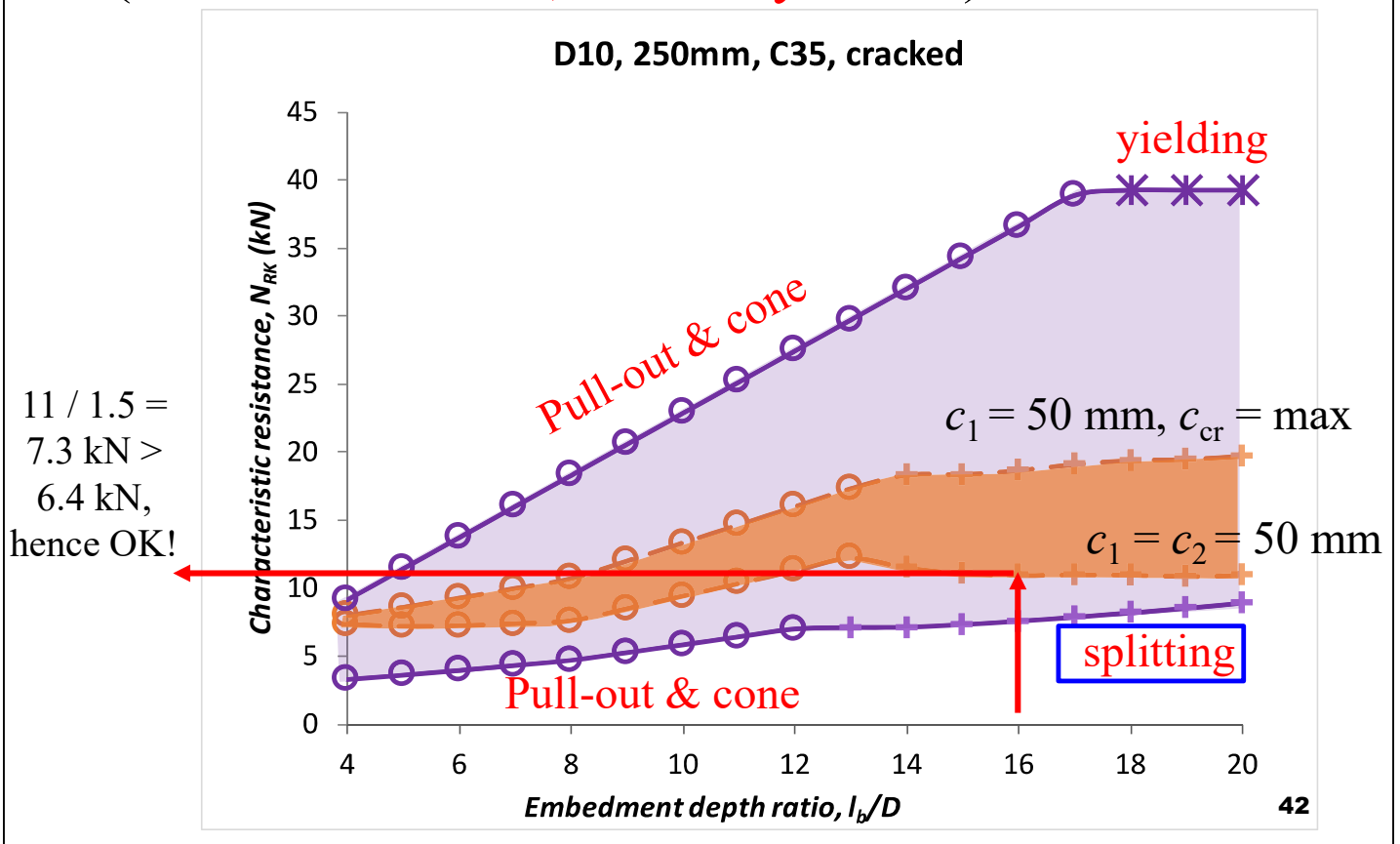
BA theory capacity graph

(uncracked concrete, no safety factor)



BA theory capacity graph

(cracked concrete, no safety factor)



Content

1. Challenges of PIR design
2. Recommendation 1: Strut-and-tie method for strut check
3. Recommendation 2: Strut-and-tie method for tie force
4. Recommendation 3: Option for bond stress
5. Recommendation 4: Minimum cover and edge distance
6. Design example
7. Reconciliation with BA theory
8. Conclusion

43

Conclusion

- The challenges of PIR design were identified:
 - 1) Very **long** anchorage length
 - 2) **Uncommon use of BA theory** in PIR practice
- 4 recommendations were proposed, anchor upon:
 - 1) **STM** for **strut check** and **tie force**
 - 2) Options for **bond strength**
 - 3) **Minimum cover** and **edge** distance
- A **design example** was illustrated
- The **BA theory** was **reconciled**.

44

1-DAY SEMINAR ON

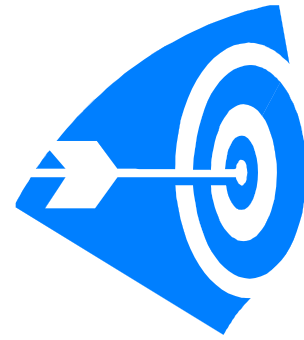
“PERFORMANCE EVALUATION FOR
CONCRETE TO CONCRETE CONNECTION:
FROM QUALIFICATION TO DESIGN”

End of Presentation on

Session 4:

Design Recommendations:

**Strut-and-Tie Method and some
reconciliations with rebar and
anchor theory**



Dr. Daniel Looi

PhD (HKU) | BEng (Malaya)

Lecturer | Swinburne University of Technology (Sarawak Malaysia)

dlooi@swinburne.edu.my